Inflation Rate = Interest Rate + Constant

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Abstract

Using consumption based asset pricing under the RUE numeraire, we show that the rate of price inflation in any currency is simply the interest rate of that currency plus a constant.

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1. Main Result

This paper is dependent on concepts introduced in the paper 'Base Currency Risk' [1] that is freely available from `http://bcr.edwardconway.co.uk` and shall be cited simply as 'BCR' for the remainder of this paper.

Denominated in the Risk Neutral RUE (RNR) numeraire (BCR [1] Section 6) consider the price of two assets: the US dollar with RNR price $\tilde{P}_\$(t)$ at time t and constant Net Convenience Yield (interest rate) $y_\$$, and, the Market portfolio consisting of all assets, with RNR price $\tilde{P}_M(t)$ at time t.

For simplicity we assume $\tilde{P}_\$(t)$ and $\tilde{P}_M(t)$ are geometric Brownian motions with constant drift, volatility and correlation as follows:

$$d\tilde{P}_\$/\tilde{P}_\$ = \tilde{\mu}_\$ dt + \tilde{\sigma}_\$ d\tilde{W}_t^\$, \tag{1}$$

$$d\tilde{P}_M/\tilde{P}_M = \tilde{\mu}_M dt + \tilde{\sigma}_M d\tilde{W}_t^M, \tag{2}$$

$$\left\langle d\tilde{W}_t^\$, d\tilde{W}_t^M \right\rangle = \tilde{\sigma}_\$ \tilde{\sigma}_M \tilde{\rho}_{\$M} dt, \tag{3}$$

where $\tilde{W}_t^\$$ and $\tilde{W}_t^M$ are Brownian motions adapted to the filtration $\mathcal{F}_t$ and $\tilde{\mu}_\$$, $\tilde{\mu}_M$, $\tilde{\sigma}_\$$, $\tilde{\sigma}_M$ and $\tilde{\rho}_{\$M}$ are constants representing the RNR drift rate of the US dollar, RNR drift rate of the Market, RNR volatility of the US dollar, RNR volatility of the Market, and the correlation between the RNR returns of the US dollar and the Market, respectively. The operator $\langle a, b\rangle$ denotes the cross-quadratic variation of a and b.

Recall (BCR [1] equation (14)) that the Market Net Income Yield is identically zero, $y_M \equiv 0$, because Net Income Yields are transfers of wealth and the Market portfolio already holds all assets.

RNR prices at time t can be calculated relative to those at some future time $T > t$ via consumption based asset pricing (BCR [1] equation (70)) resulting in the following equations:

$$\tilde{P}_\$(t) = \beta(t,T) \cdot \mathbb{E}_t[\tilde{P}_\$(T)] \exp\left((T-t) y_\$\right), \tag{4}$$

$$\tilde{P}_M(t) = \beta(t,T) \cdot \mathbb{E}_t[\tilde{P}_M(T)], \tag{5}$$

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where $\beta(t,T)$ is a function representing impatience that arises from consumption based asset pricing.

The time t US dollar denominated price of the Market portfolio, $E_{M\$}(t)$, is therefore given by

$$E_{M\$}(t) = \frac{\tilde{P}_M(t)}{\tilde{P}_\$(t)}. \tag{6}$$

Using equations (4) and (5) we see that the drift rate of the US dollar asset and the drift rate of the Market portfolio, both denominated in RNR from time t to time T, are given by

$$\tilde{\mu}_\$ = \frac{1}{T-t}\log\frac{\mathbb{E}_t[\tilde{P}_\$(T)]}{\tilde{P}_\$(t)} = -y_\$ - \frac{1}{T-t}\log\beta(t,T), \tag{7}$$

$$\tilde{\mu}_M = \frac{1}{T-t}\log\frac{\mathbb{E}_t[\tilde{P}_M(T)]}{\tilde{P}_M(t)} = -\frac{1}{T-t}\log\beta(t,T). \tag{8}$$

So $\tilde{\mu}_M - \tilde{\mu}_\$ = y_\$$. Via Itô, the drift rate $\mu_{M\$}$ of all assets denominated in US dollars is

$$\mu_{M\$} = \mathbb{E}_t\left[\frac{dE_{M\$}}{E_{M\$}}\right]/dt, \tag{9}$$

$$= \tilde{\mu}_M - \tilde{\mu}_\$ + \tilde{\sigma}_\$^2 - \tilde{\sigma}_M\tilde{\sigma}_\$\tilde{\rho}_{M\$}, \tag{10}$$

$$= y_\$ + k, \tag{11}$$

where $k := \tilde{\sigma}_\$^2 - \tilde{\sigma}_M\tilde{\sigma}_\$\tilde{\rho}_{M\$}$ is a constant.

So if we define the rate of US dollar inflation to be the drift rate $\mu_{M\$}$ of the US dollar price of all assets, then the rate of inflation is equal to the US dollar interest rate $y_\$$ plus the constant k. As nothing special has been assumed about the US dollar asset, an equivalent result holds for any currency.

An intuitive explanation for this result is that in an efficient market such as that modelled here, carry trades cannot be a free lunch. The valuation of any currency will thus tend to drift towards its forward price with time. The higher interest rates are, the lower the forward price will be, and as the value of a currency drifts lower we experience inflation.

Interest rates are therefore the fundamental driver of inflation. Note that if real interest rates are positive then $k < 0$. Zero interest rates would then be expected to result in deflation at a rate equal to the usual level of real interest rates.

References

[1] E. Conway, Base currency risk (2015).
URL `http://bcr.edwardconway.co.uk`